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SPECIAL BRIEF NOTE

FLUID FORCES AND DYNAMICS OF A HYDROELASTIC STRUCTURE WITH VERY LOW MASS AND DAMPING

A. KHALAK and C. H. K. WILLIAMSON

Mechanical and Aerospace Engineering, Upson Hall, Cornell University, Ithaca, NY 14853, U.S.A.

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In this paper we present some central new results from a study of the dynamics and fluid forcing on an elastically mounted rigid cylinder, constrained to oscillate transversely to a free stream. With very low damping, and with a low specific mass that is around 1% of the value used in the classic study of Feng (1968), we show that the cylinder excitation regime extends over a large range of normalized velocity (around four times that found by Feng), with a large amplitude which is around twice that of Feng. Four distinct regions of response are identified, namely the initial excitation region, the "upper branch" (of very high amplitude response), the "lower branch" (of moderate amplitude response), and the desynchronization region. There are distinct differences in the character of mode transitions, as follows. As normalized velocity is increased, there is a hysteretic jump from an initial excitation regime to the upper branch, whereas the jump from the upper to the lower branch involves an intermittent switching, which is illustrated by plotting the instantaneous phase between lift force and displacement using the Hilbert transform. Contrary to classical "lock-in", whereby the oscillation frequency matches the structural natural frequency, we find that the oscillation frequency increases markedly above the natural frequency, through the excitation regime. Finally, we present the first lift force measurements for such a freely vibrating cylinder experiment, yielding a maximum lift coefficient of around 4·5, whereas a maximum drag coefficient of 6·0 is also measured. The lift is comparable, but somewhat higher, than the forces measured ($C_L \sim 2{\cdot}0$) in the equivalent free-vibration experiments of Hover *et al.* (1997), involving force-feedback and on-line computer-simulation of the modelled structure. Both the lift and drag maxima exhibit at least a five-fold increase over the stationary cylinder case. Perhaps the largest effect is found for the fluctuating drag, which is found to be upto 100 times that measured for a static cylinder.

1. INTRODUCTION

THE PROBLEM of vortex-induced vibration of a cylinder, in particular the case where a rigid circular cylinder is elastically mounted and constrained to oscillate transversely to a free stream, has been well-studied in the literature, as may be seen from the comprehensive reviews of Sarpkaya (1979), Bearman (1984) and Parkinson (1989). However, there remain some rather basic questions concerning vibration phenomena under the conditions of very low mass and damping, and for which there are almost no laboratory investigations. As one reduces the mass ratio [m^* = (oscillating mass) /(displaced fluid mass)] to 1% of the value used in the classical study of Feng (1968), it is of significant and fundamental interest to know what is the dominant response frequency during excitation; what is the range of

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normalized velocity for significant oscillations or "lock-in", and perhaps the most basic of all the questions: what is the amplitude of response as a function of normalized velocity? We address these questions in the present paper.

As a central component of this paper, we address the question of fluid forcing on a freely vibrating structure, which has received surprisingly little attention since the early displacement measurements of Feng (1968), and those of the more recent aeroelastic study of Brika & Laneville (1993). Despite the extensive force measurements for a cylinder undergoing transverse forced vibration [see, for example, Sarpkaya (1978), Staubli (1983), Gopalkrishnan (1993), and others], there have appeared no direct lift-force measurements in the literature for such an elastically mounted arrangement. One should note, however, that Sarpkaya (1995) recently presented a set of drag measurements for a cylinder which can oscillate both in-line and transverse to the flow. Such a two-degree-of-freedom arrangement, as pioneered by Sarpkaya, should yield very practical and significant new results, and several groups are now working on this problem. We should also note that Hover *et al.* (1997), in conjunction with the research group of Michael Triantafyllou at MIT, have developed an ingenious experiment whereby a cylinder is made to oscillate as if it were free, by simulating the mass-spring-damper in real time on a computer, and by using also as input the instantaneous force measured on the cylinder. In the present work, we shall present new force measurements of lift and drag for a hydroelastic cylinder of very low mass and damping.

On a final point in this introduction, we may mention the extensive work that has been done to predict vortex-induced vibration using nonlinear oscillator models, which are well reviewed in the paper by Parkinson (1989). An excellent example is that of Cui & Dowell (1981), who discuss an approximate method to predict oscillations of bluff bodies in air and in water, discussing [as later in Bearman (1984)] the fact that the regime of response in water is typically greater than in air. A full comparison between such a predictive approach with extensive experiments is perhaps still needed. However, in the present brief presentations, this is not possible.

2. EXPERIMENTAL DETAILS

In our hydroelastic studies with very low mass and damping, we have constructed a unique apparatus, which operates in conjunction with the Cornell-ONR Water Channel. One may refer to Khalak & Williamson (1996) for the details concerning this experimental arrangement, and only the essential outline of the experimental set-up will be mentioned here. An air-bearing facility, allowing only motion transverse to the flow, is located atop the water channel, and supports a vertical cylinder in the flow. A two-axis force balance utilizing LVDTs can measure lift and drag *simultaneously* with the measurement of displacement, with the latter using a non-contact (magnetostrictive) position transducer. These pieces of equipment are part of the overall oscillating mechanism above the water surface. Two test cylinders are used of diameters 1·5 in. and 2·0 in. (1 in. = 25·4 mm), giving length-diameter ratios of 10 and 8.5, respectively. Our set-up has an order of magnitude lower mass–damping parameter ($m^*\zeta$ = mass ratio $\times$ damping ratio) than previous experiments of this type, as shown in Khalak & Williamson (1996).

The normalized velocity in this study is given the symbol U^*, where $U^* = U/f_nD$, with U being the free-stream velocity, f_n the natural frequency of the structure in water, and D the cylinder diameter. Drag force coefficient is denoted as C_x, and lift as C_y, with root-mean-square or maximum values defined with the terminology "r.m.s." or "max", respectively.

Force coefficients are normalized by the conventional product $\frac{1}{2}\rho U^2 DL$, where L is the cylinder length and ρ the fluid density. Normalized position amplitude is given as A = amplitude/diameter, and frequency ratio is f^* = (oscillation frequency)/(natural frequency). The overall nondimensional quantities in this experiment, that we shall discuss here, are thus $\{U^*, m^*, \zeta, A, f^*\}$.

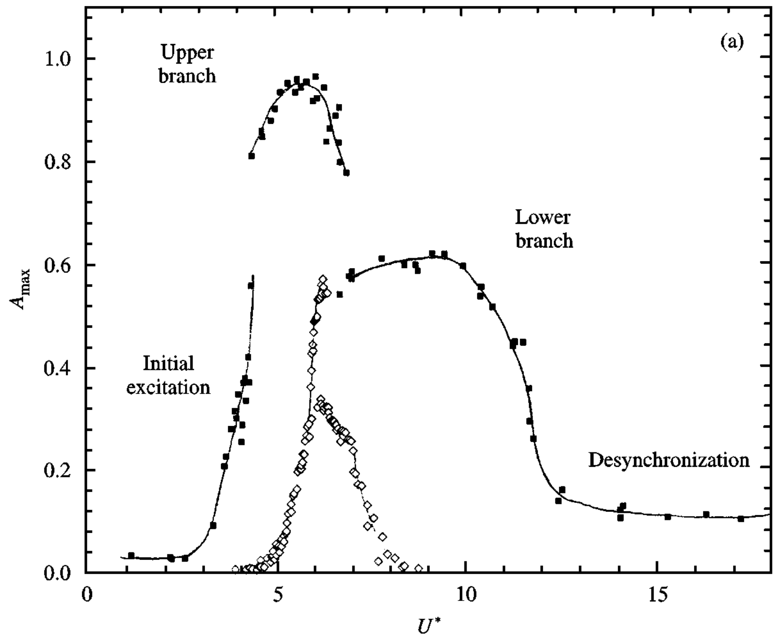
3. AMPLITUDE AND FREQUENCY RESPONSE FOR LOW MASS AND DAMPING

We may compare, in Figure 1(a), our amplitude response plot for $m^* = 2{\cdot}4$ and $\zeta = 0{\cdot}0045$ to the response measured by Feng (1968), in whose experiments $m^* = 248$. Since the present results have much lower mass–damping ($m^*\zeta$) than those of Feng (3% of his value), we find much greater amplitudes, with nearly twice the maximum amplitude ($A \sim 1$). We also find a four-fold increase in the range of velocity U^* over which the cylinder is excited. In our low mass–damping experiments, we shall broadly characterize in Figure 1(a) four regimes, as (i) the initial excitation regime; (ii) the "upper" branch of response; (iii) the "lower" branch of response; and (iv) the desynchronization regime. Interestingly, the initial excitation regime (see later in Figure 4), exhibits two branches of its own, when $A_{\max}$ is plotted. This phenomenon is associated with two "sub-regimes", whereby at lower U^* the response is distinctly *quasiperiodic*, while for the slightly higher U^*, the response becomes abruptly more *periodic*, with a single-frequency peak. [Further details of these intricate branches are included in Khalak & Williamson (1997b)]. In brief, as one increases U^*, we find that the amplitude jumps up to the upper branch, and then intermittently switches between the upper and lower branches, before finally dropping out of synchronization into the decoherence regime. These transitions will be discussed somewhat more in Section 5.

A particularly surprising result is found in Figure 1(b), where the oscillation frequency follows neither the vortex-shedding frequency (for the stationary cylinder case), nor the structural natural frequency. In classical "lock-in" from vortex-induced vibrations (see the review papers mentioned earlier), the oscillation frequency "locks onto" the natural frequency. It is thus clear that the conditions of low mass and damping exhibit quite nonclassical phenomena. It was pleasing to find an almost equivalent frequency phenomenon occurring for another vortex-induced vibration experiment involving the dynamics of a tethered sphere in a flow, also with very low mass–damping (Govardhan & Williamson 1997). Similar results are being found for an elastically mounted cylinder by Tony Leonard's research group at Caltech, in both computations (with Doug Shiels), and also in two-degree-of-freedom experiments (with Mo Gharib). It thus seems that this type of nonclassical frequency response is a real and most interesting feature of very low mass–damping structures.

4. FORCE MEASUREMENTS

Force measurements were made for $m^* = 10{\cdot}1$ and $\zeta = 0{\cdot}00134$, as shown in Figure 2. The drag forces measured during such a free-vibration experiment are quite massive, reaching a maximum $C_{x_{\max}}$ of 5·2, with mean C_x of around 3·5. This is a three-fold increase over the stationary case, but the difference between the maximum and mean drag values also indicates the enormous drag-force fluctuations, which we will subsequently mention further. The lift force $C_{y_{\mathrm{rms}}}$ reaches 2·0, which corresponds to a maximum C_y near 3, suggesting a five-fold increase in r.m.s. lift over the stationary cylinder case. Both the drag and lift



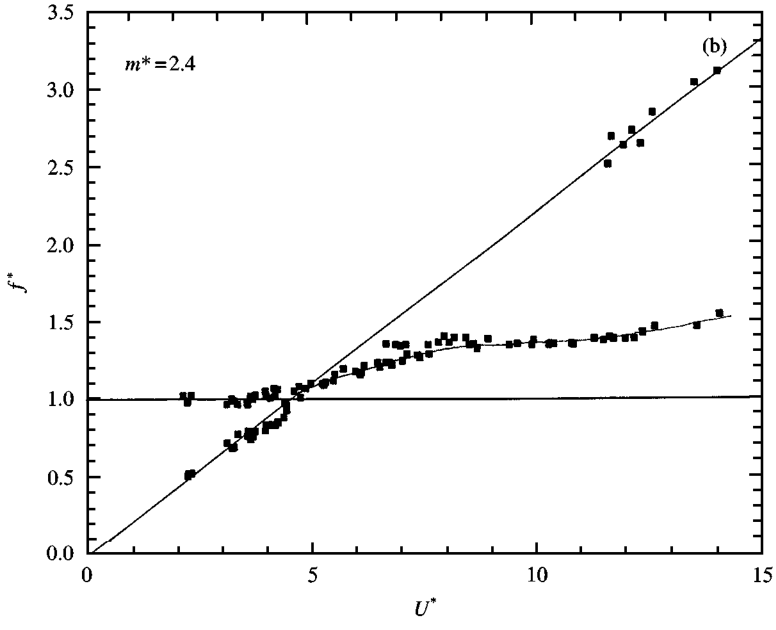


Figure 1. Response amplitude and frequency for low mass and damping. In (a), we compare the amplitude response between the present study, with $m^* = 2{\cdot}4$, with that of Feng (1968), who used $m^* = 248$. Note the four-fold increase in the range of U^* for excitation, and the increased amplitude of the present study. (◇) Feng (1968); (■) present study. In (b), we present oscillation frequency data through the excitation regime, indicating the nonclassical result that the frequency is significantly above the natural frequency line, while at the same time it is below the vortex shedding frequency (of the non-oscillating case).

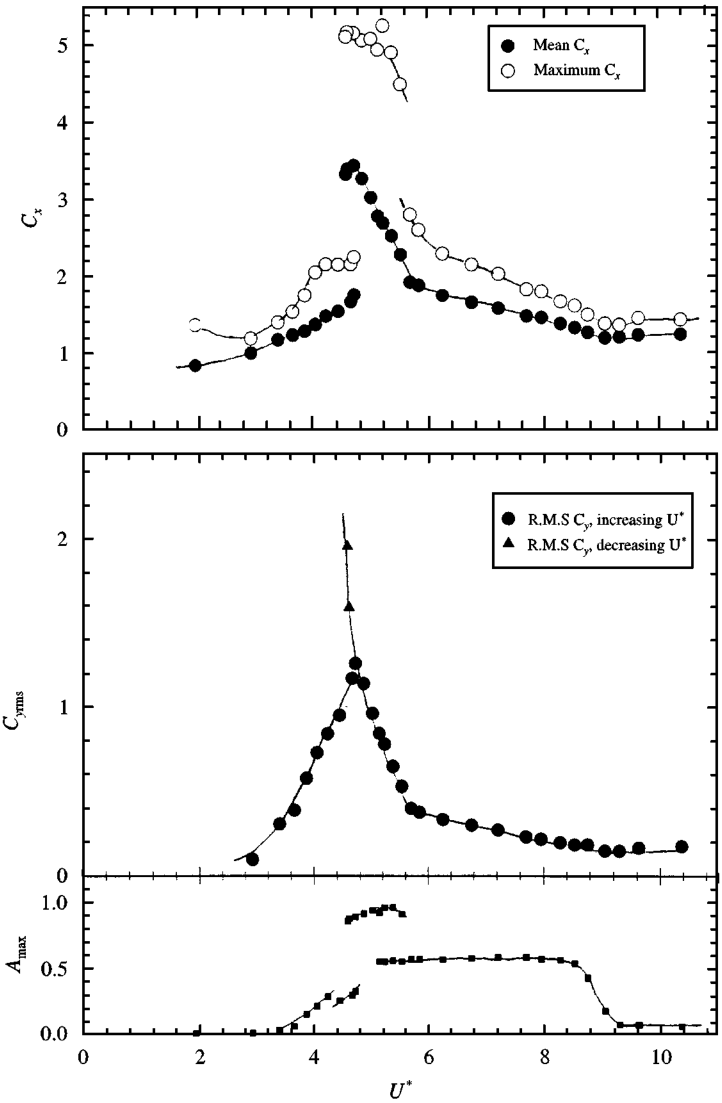


Figure 2. Variation of drag and lift force with normalized velocity, for $m^* = 10{\cdot}1$. In the case of the lift force, points taken with increasing velocity are plotted with circles, whereas for decreasing velocity they are triangles. The plot of A_{max} is given for reference.

rapidly increase through the initial excitation regime, they peak at the beginning of the upper-branch region, they decrease rapidly throughout the upper branch, and then gradually diminish through the lower-branch region. The most striking feature of the r.m.s. lift plot is the very sharp peak, which occurs right at the transition between initial excitation region and the upper branch. Before the transition, the lift is rapidly increasing, while after the transition in the different mode of response the lift is rapidly decreasing, thus yielding a very sharp lift peak. Surprisingly, the lift in the lower branch is very low, even smaller than the stationary case at equivalent Reynolds numbers (around 6000).

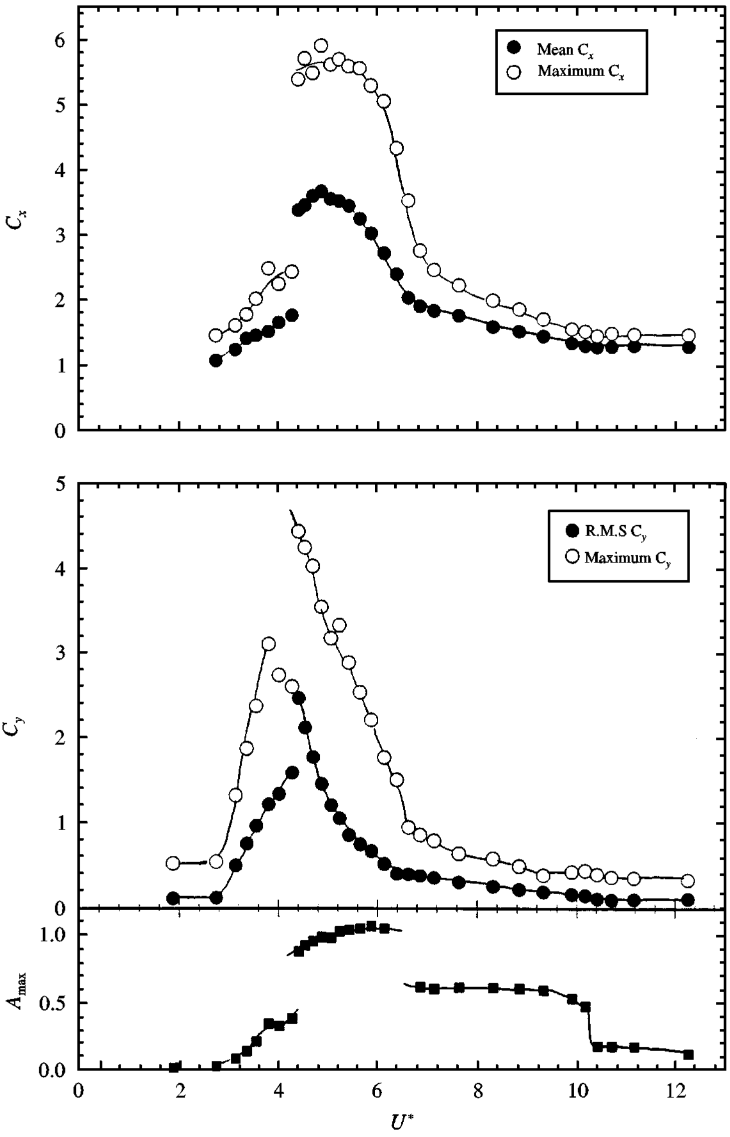


Figure 3. Variation of drag and lift force with normalized velocity, for $m^* = 3{\cdot}3$. The plot of A_{max} is given for reference.

Force measurements were also made at a lower mass ratio, of $m^* = 3{\cdot}3$, at a value $\zeta = 0{\cdot}00260$, making the mass–damping $m^*\zeta = 0{\cdot}00858$ (smaller than the value $m^*\zeta = 0{\cdot}0135$ used for the case $m^* = 10{\cdot}1$). It is shown comprehensively in Khalak & Williamson (1997a) that the response amplitudes increase as the parameter $m^*\zeta$ is reduced (even in our present regime of very low $m^*\zeta$). We also find here that the induced forces increase as $m^*\zeta$ is reduced, and the mean drag becomes 3·8, with the maximum value reaching $C_{x_{max}} = 6{\cdot}0$, yielding a five-fold increase over the static case. Likewise, the lift

$C_{y_{rms}}$ shows a peak of 2·5, at the transition between initial excitation and the upper branch, and a maximum lift $C_{y_{max}}$ of 4·5.

The forced vibration of a structure can yield larger fluid-induced forces than for fixed cylinders, a result which is well known from past work referenced in the reviews mentioned earlier, although the extent of the increase is certainly very large. In the case of biharmonic free vibrations, involving simultaneous oscillations in-line and transverse to the flow, Sarpkaya (1995) finds that the mean drag for an oscillating cylinder can become 3·5 times that for the static cylinder, which is comparable to values found here. He also gives a mean drag around three times the static case, if the cylinder is forced into transverse vibration (Sarpkaya 1978). The very recent work of Hover *et al.* (1997), using their ingenious equivalent free-vibration experiment, gives a r.m.s. lift coefficient of roughly 1·6 – 2·0, when the mass–damping is set to zero ($m^*\zeta = 0$), which is comparable to the present values (we computed their value from their power-spectral density of the signal, taken near the lift frequency). However, we expect to measure, in ongoing experiments, somewhat larger lift forces as $m^*\zeta$ becomes very small (tending to zero), based on the present measurement trends.

We therefore find, at least in our experiments to date, a five-fold increase in the maximum drag, and a six-fold increase in the r.m.s. lift force, but perhaps the most impact is given by considering the increase in fluctuating drag. For a static cylinder at Reynolds number of 10 000, Gopalkrishnan (1993) gives a fluctuating drag which is around 2% of its mean value. For our $m^* = 3{\cdot}3$ case above, the fluctuating drag is around 60% of the mean value, which gives a fluctuating drag coefficient (2·2) around 102 times what is measured for the stationary cylinder case! This compares well with the typical fluctuating drag coefficients (2·0) measured from forced oscillation experiments by Gopalkrishnan (1993), for $A/D \sim 0{\cdot}75$, and at the same normalized frequencies.

5. TRANSITIONS BETWEEN RESPONSE-AMPLITUDE BRANCHES

It is of particular interest to investigate the manner in which the vibrating structure can have a transition between one mode of response and another, and we focus here on the transition from initial excitation to the upper branch, and the transition from the upper branch to the lower branch. These transitions are illustrated in Figure 4, where in (a) we find a distinct hysteresis between initial excitation and the upper branch, as evidenced from response amplitude data. As U^* is increased, the jump up to the high amplitude occurs around $U^* = 4{\cdot}70$, whereas for decreasing velocity the drop down to the initial excitation regime occurs at close to $U^* = 4{\cdot}50$.

The second transition discussed here is, however, rather different to the first. Although there are distinctly different conditions pertaining to the upper branch versus the lower branch (different amplitudes, phase between lift and displacement, and frequency, among other parameters), the system switches between these branches intermittently, and seems only weakly "locked" into one or the other mode over the range $U^* = 5{\cdot}0$–$5{\cdot}6$. The intermittent switching is demonstrated by applying the Hilbert transform to lift force–displacement data, and this technique is described further in Khalak & Williamson (1997b). The main results relevant here are shown in Figure 4(b). If the phase ϕ is defined as the lead of the lift-force fluctuations over the displacement, then the upper branch ϕ is near 0°, whereas the lower branch phase is near 180°, rather like what is found for a (low-damped) linear system going through resonance. What is interesting is that the phase, which is directly indicative of the predominant mode of oscillation (upper or lower-branch type of

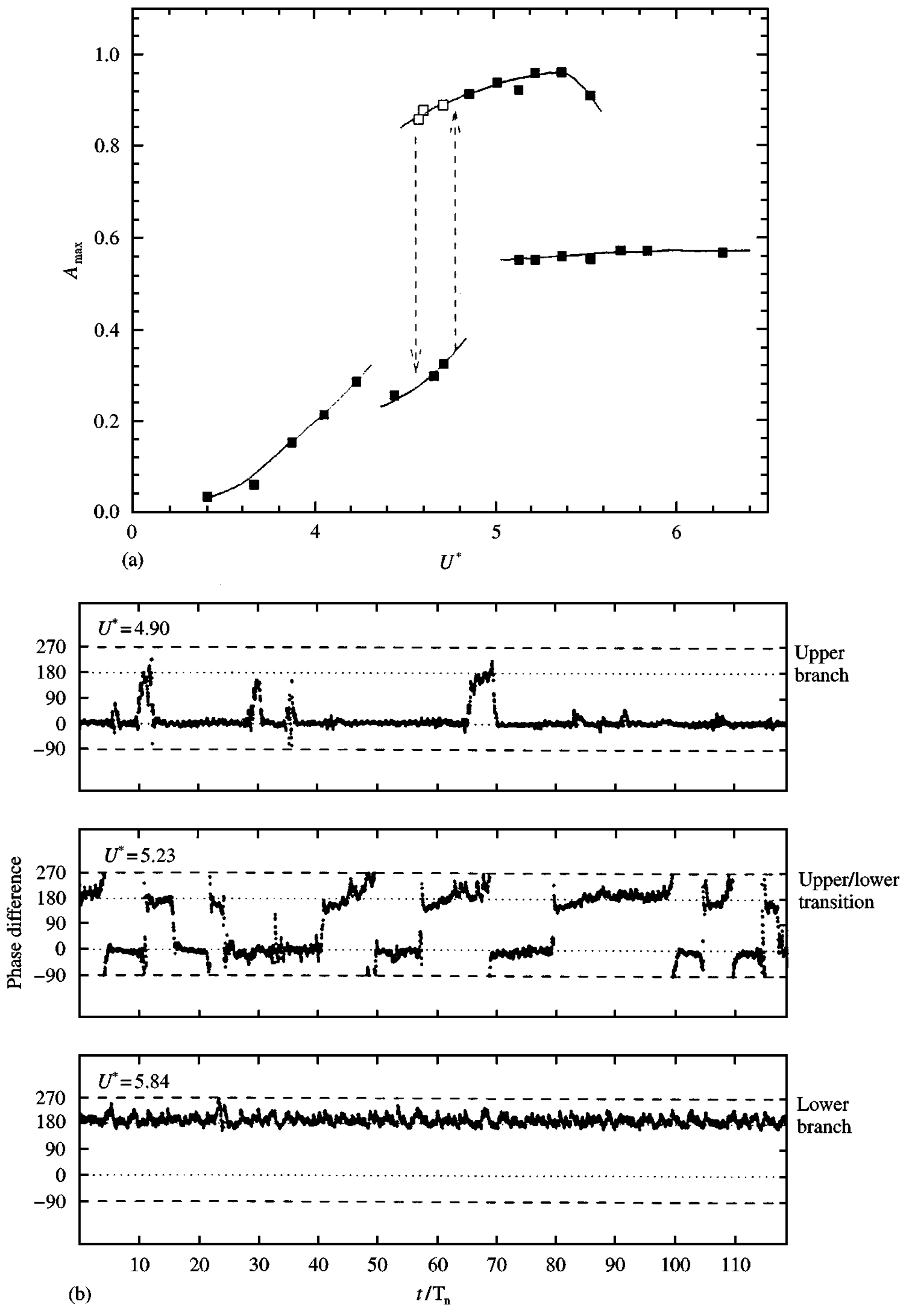


Figure 4. Character of the two transitions. In (a), for the first transition, a hysteretic jump is demonstrated, from the initial excitation region to the upper branch. In (b), for the second transition, an intermittent switching is found, between the upper branch mode and the lower branch mode, as evidenced by the intermittent switching of the phase between lift and displacement. This is the first measurement of instantaneous phase for this problem, and is made possible by utilizing the Hilbert transform. (Other indicators such as instantaneous amplitude or frequency equally provide evidence for a corresponding switching of the modes.)

oscillation), seems to remain for a number of periods hovering around one mode, and then to (sometimes abruptly) switch to the other mode, and this scenario is clearly apparent in the central phase plot of Figure 4(b) when $U^* = 5{\cdot}23$. Therefore, whereas the first transition up to the upper branch is distinctly hysteretic, the second transition down to the lower branch does involve a jump (as for the first), but the modes are found to switch intermittently. As velocity U^* is increased, the amount of time spent on the upper-branch mode is decreasing, while the time spent on the lower-branch mode is increasing, until, at sufficiently high U^* (see the figure for $U^* = 5{\cdot}84$), the mode is stably the lower-branch type all of the time.

6. CONCLUSIONS

In the quest to investigate vortex-induced vibrations of a structure at very low mass and damping, we have designed an experiment which yields a large regime of normalized velocities over which the cylinder is significantly excited, and for which amplitudes of one diameter are measured, both of which are markedly greater than the classical response found for high mass ratio by Feng (1968). The amplitude response plot is marked by two branches, with distinctly different levels of response, namely the "upper branch" and "lower branch". The transition from initial excitation to the upper branch is hysteretic, while the second transition from upper branch to lower branch is the one involving an intermittent switching between the modes of oscillation. The intermittency of the second transition is evident from instantaneous phase measurements between lift force and displacement, although other indicators, such as the instantaneous frequency or instantaneous amplitude [also found using the Hilbert transform described in Khalak & Williamson (1997b)] also clearly exhibit the intermittency. This study appears to present the first simultaneous lift-force measurements on such a freely vibrating experiment, yielding large augmentation of the lift over the stationary cylinder case. The r.m.s. lift experiences a six-fold increase, whereas the maximum drag has a five-fold increase, but perhaps the most impressive increase is the 100 times increase for the fluctuating drag coefficient, as compared with the stationary cylinder values at similar Reynolds numbers.

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